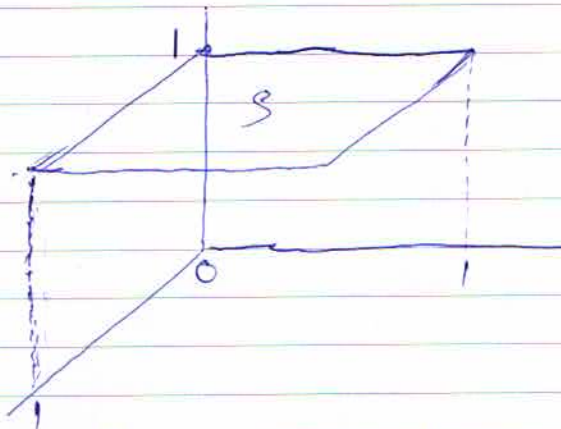


$$\underline{v} = yz \underline{i} + 3xz \underline{j} + z \underline{k}$$

$$\nabla \times \underline{v} = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 3xz & z \end{bmatrix} = -3x \underline{i} + y \underline{j} + (3z - z) \underline{k}$$

$$= -3x \underline{i} + y \underline{j} + 2z \underline{k}$$



$$\int_S \nabla \times \underline{v} \cdot \underline{n} \, ds = \oint_C \underline{v} \cdot \underline{k} \, dl$$

$$\nabla \times \underline{v} \cdot \underline{n} \, ds = \nabla \times \underline{v} \cdot \underline{k} \, dx \, dy = 2z$$

$$\int_S \nabla \times \underline{v} \cdot \underline{n} \, ds = \int_0^1 \int_0^1 2(1) \, dx \, dy = 2(1)(1) = 2$$

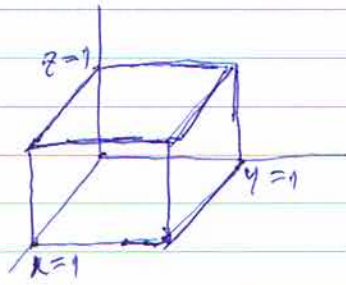
$$\oint_C \underline{v} \cdot \underline{k} \, dl = \int_0^1 \int_0^1 3xz \, dy + \int_0^1 \int_0^1 (-yz) \, dx + \int_0^1 \int_0^1 (-3xz) \, dy + \int_0^1 \int_0^1 yz \, dx$$

$$= \int_0^1 3 \, dy - \int_0^1 1 \, dx = 2$$

$$\int_S \nabla \times \underline{v} \cdot \underline{n} \, ds = \oint_C \underline{v} \cdot \underline{k} \, dl = 2 \quad \#$$

Note

$$\underline{v} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$$



$$\nabla \cdot \underline{v} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(2y) + \frac{\partial}{\partial z}(3z) = 6$$

$$\int_V \nabla \cdot \underline{v} dV = \int_0^1 \int_0^1 \int_0^1 6 dx dy dz = 6$$

$$\begin{aligned} \oint_S \underline{v} \cdot \underline{n} ds &= \int_0^1 \int_0^1 \underline{v} \cdot \hat{i} dy dz + \int_0^1 \int_0^1 \underline{v} \cdot (-\hat{i}) dy dz + \int_0^1 \int_0^1 \underline{v} \cdot \hat{j} dx dz \\ &\quad + \int_0^1 \int_0^1 \underline{v} \cdot (-\hat{j}) dx dz + \int_0^1 \int_0^1 \underline{v} \cdot \hat{k} dx dy + \int_0^1 \int_0^1 \underline{v} \cdot (-\hat{k}) dx dy \\ &= \int_0^1 \int_0^1 (1) dy dz + \int_0^1 \int_0^1 2(1) dx dz + \int_0^1 \int_0^1 3(1) dx dy \\ &\quad 1 + 2 + 3 = 6 \end{aligned}$$

∮_S $\underline{v} \cdot \underline{n} ds$

$$\int_V \nabla \cdot \underline{v} dV = \oint_S \underline{v} \cdot \underline{n} ds = 6$$

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Note